WHAT IS CLAIMED IS:

1. A method for characterizing distortions in the earth's magnetic field caused by a vehicle, a magnetometer affixed to the vehicle, said method comprising:

repeatedly measuring the distorted magnetic field utilizing the magnetometer;

obtaining a three-dimensional orientation of the vehicle axes with respect to the earth at a time of each magnetometer measurement;

receiving undistorted earth magnetic field data for the vicinity of the vehicle relative to the earth at the time of each magnetometer measurement; and

utilizing the magnetic field measurements, the orientations of the vehicle, and the undistorted earth magnetic field data to characterize distortions caused by one or more of the vehicle and magnetometer errors.

2. A method according to Claim 1 wherein obtaining an orientation of the vehicle axes comprises receiving pitch and roll signals from one or more of an attitude heading reference system, an attitude reference system, an inertial reference system, and an inertial reference unit.

3. A method according to Claim 2 wherein obtaining an orientation of the vehicle axes comprises:

moving a vehicle equipped with a GPS;

determining a GPS tracking angle; and

assuming the GPS tracking angle is the true heading.

4. A method according to Claim 2 wherein obtaining an orientation of the vehicle axes comprises:

moving a vehicle equipped with a GPS; and

calculating a true heading during, and immediately following, vehicle turns or accelerations, utilizing GPS-augmented attitude and heading reference system algorithms.

5. A method according to Claim 1 wherein obtaining an orientation of the vehicle axes comprises receiving pitch, roll, and at least one of a true heading signal and a synthetic magnetic heading signal from an inertial reference system.

6. A method according to Claim 1 wherein obtaining an orientation of the vehicle comprises:

receiving pitch and roll signals from an attitude heading reference system; and

receiving at least one of a true heading signal and a synthetic magnetic heading signal from an inertial reference system.

7. A method according to Claim 1 wherein obtaining an orientation of the vehicle comprises:

receiving pitch and roll signals from an attitude heading reference system; and

determining a true heading utilizing a dual antenna GPS.

8. A method according to Claim 1 wherein receiving undistorted earth magnetic field data comprises determining vehicle position and using a table of magnetic inclination and declination data for a surface of the earth.

9. A method according to Claim 1 wherein characterizing distortions caused by the vehicle comprises transforming undistorted magnetic field components to distorted magnetic field components over relative orientations between

the undistorted magnetic field and one or more of the axes of the vehicle and the axes of the magnetometer.

10. A method according to Claim 1 wherein obtaining a three-dimensional orientation of the vehicle axes comprises determining vehicle position using at least one of an inertial navigational system and ground based navigation aids.

11. A method according to Claim 1 wherein characterizing distortions comprises calculating correction coefficients for the magnetometer.

12. A method according to Claim 11 wherein calculating correction coefficients comprises compensating the magnetometer by estimating $\mathbf{L}_e$ and $\mathbf{H}_{pe}$ in the function $\mathbf{H}_{earth} = \mathbf{L}_e * \mathbf{H}_{meas} + \mathbf{H}_{pe}$ where, $\mathbf{H}_{earth}$ is a three-dimensional vector representing the undisturbed Earth's magnetic field, $\mathbf{H}_{meas}$ represents a three-dimensional vector for the disturbed Earth's magnetic field as measured by magnetometer, $\mathbf{L}_e$ is a three by three (3x3) matrix of magnetic correction coefficients, and $\mathbf{H}_{pe}$ is a three-dimensional vector of magnetic correction coefficients.

13. A method of compensating a magnetometer affixed to a vehicle to obtain accurate magnetic heading information for a vehicle orientation, said method comprising

using the magnetometer to measure a distorted earth magnetic field relative to axes of the vehicle;

determining a pitch and roll orientation of the vehicle axes with respect to the earth;

calculating the distortion of the earth's magnetic field for any relative angle between the vehicle axes and the earth's undistorted magnetic field; and

determining a magnetic heading based on the magnetometer measurement, adjusted by the pitch and roll orientation of the vehicle, and compensated for distortions of the earth's magnetic field.

14. A method according to Claim 13 wherein determining a magnetic heading further comprises compensating the magnetometer by solving the function $\mathbf{H}_{earth} = \mathbf{L}_e * \mathbf{H}_{meas} + \mathbf{H}_{pe}$ by estimating $\mathbf{L}_e$ and $\mathbf{H}_{pe}$ where, $\mathbf{H}_{earth}$ is a three-dimensional vector representing the undisturbed Earth's magnetic field, $\mathbf{H}_{meas}$ represents a three-dimensional vector for the disturbed Earth's magnetic field as measured by magnetometer, $\mathbf{L}_e$ is a three by three (3x3) matrix of magnetic correction coefficients, and $\mathbf{H}_{pe}$ is a three-dimensional vector of magnetic correction coefficients.

15. A method according to Claim 13 further comprising augmenting pitch and roll orientation data based on magnetometer measurements and the magnetometer compensation.

16. A method according to Claim 13 further comprising estimating $\mathbf{L}_e$ and $\mathbf{H}_{pe}$ based on multiple measurements of $\mathbf{H}_{earth}$ and $\mathbf{H}_{meas}$.

17. A method according to Claim 16 wherein determining a magnetic heading comprises:

determining vehicle position and using a table of magnetic inclination and declination data for a surface of the earth;

comparing the measured magnetic field, $\mathbf{H}_{meas}$, against the magnetic inclination and declination data to determine the magnetic correction coefficients, $\mathbf{H}_{pe}$; and

providing $\mathbf{L}_e$ and $\mathbf{H}_{pe}$ as correction coefficients to the magnetometer.

18 A method according to Claim 13 further comprising determining the vector of magnetic correction coefficients, $\mathbf{H}_{pe}$, and the matrix of magnetic correction coefficients, $\mathbf{L}_e$, utilizing a three-dimensional magnetic error table, the error table configured to maintain separate data points for multiple aircraft orientations between an actual magnetic field and vehicle pitch, roll, and yaw axes.

19. A method according to Claim 18 wherein the magnetic error table comprises separate data points at 30 degree increments in yaw and at 60 degree

increments in pitch and roll, each data point being a three-dimensional vector of magnetic error relative to vehicle axes.

20. A method according to Claim 19 comprising determining the correction coefficients utilizing scalar values from the data points.

21. A method for determining a true earth magnetic field from a magnetic field measured by a magnetometer, said method comprising:

generating a truth reference field vector, $\tilde{\mathbf{h}}_i$, from sources of pitch, roll, heading, and position independent of the magnetometer and a three dimensional map of the earth's magnetic field;

determining a difference between a vector as measured by the magnetometer and the truth reference vector; and

utilizing the difference to estimate corrections to magnetometer model coefficients.

22. A method according to Claim 21 further comprising utilizing the difference to estimate corrections to one or more of vehicle pitch, roll, and heading.

23. A method according to Claim 21 wherein pitch, roll, and heading data is provided by at least one of a GPS, an attitude and heading reference system, an inertial reference system, an inertial navigation system, and ground based navigational aids.

24. A method according to Claim 21 further comprising utilizing the difference to provide corrections to attitude and heading angles of a navigation system.

25. A method according to Claim 21 further comprising estimating corrections using a Kalman filter.

26. A method according to Claim 21 wherein generating a truth reference field vector comprises relating the field measured by the magnetometer $\tilde{\mathbf{h}}_m$ to the true earth's field $\tilde{\mathbf{h}}$ according to $\tilde{\mathbf{h}}_m = \mathbf{M}\tilde{\mathbf{h}} + \tilde{\mathbf{h}}_p$, where M is a 3x3 magnetic permeability matrix and $\tilde{\mathbf{h}}_p$ is a 3x1 vector of field offset errors resulting from permanent magnetization.

27. A method according to Claim 26 wherein determining a difference comprises:

solving for the true earth's field, $\tilde{\mathbf{h}}$, according to $\begin{aligned}\tilde{\mathbf{h}} &= \mathbf{M}^{-1}\left(\tilde{\mathbf{h}}_m - \tilde{\mathbf{h}}_p\right)\\ &= \mathbf{L}\left(\tilde{\mathbf{h}}_m - \tilde{\mathbf{h}}_p\right)\end{aligned}$,

where the matrix $\mathbf{L}$ is defined as $\mathbf{L} = \mathbf{M}^{-1}$; and

assuming that the initial value of matrix $\mathbf{L}$ is equal to the identity matrix and the initial vector of field offset errors, $\tilde{\mathbf{h}}_p$, is zero.

28. A method according to Claim 27 further comprising:

obtaining a linearized error equation for the magnetometer-measured and compensated body-axis earth's field components according to $\begin{aligned}\delta\tilde{\mathbf{h}} &= \delta\mathbf{L}\left(\tilde{\mathbf{h}}_m - \tilde{\mathbf{h}}_p\right) - \mathbf{L}\delta\tilde{\mathbf{h}}_p\\ &= \delta\mathbf{L}\Delta\tilde{\mathbf{h}} - \mathbf{L}\delta\tilde{\mathbf{h}}_p\end{aligned}$, where $\Delta\tilde{\mathbf{h}}$ is defined as $\Delta\tilde{\mathbf{h}} = \tilde{\mathbf{h}}_m - \tilde{\mathbf{h}}_p$; and

forming a separate measurement equation for each field component from the linearized error equation according to $\delta\tilde{h}_x = \Delta\tilde{\mathbf{h}}^T\delta\mathbf{l}_{r1} - \mathbf{l}_{r1}^T\delta\tilde{\mathbf{h}}_p$, $\delta\tilde{h}_y = \Delta\tilde{\mathbf{h}}^T\delta\mathbf{l}_{r2} - \mathbf{l}_{r2}^T\delta\tilde{\mathbf{h}}_p$, and $\delta\tilde{h}_z = \Delta\tilde{\mathbf{h}}^T\delta\mathbf{l}_{r3} - \mathbf{l}_{r3}^T\delta\tilde{\mathbf{h}}_p$, where each equation is in the form of a row vector multiplied by a column vector of magnetometer parameter errors to provide scalars that are functions of the vectors.

29. A method according to Claim 28 where magnetometer errors are provided according to $\delta\tilde{\mathbf{h}} = \begin{bmatrix} \Delta\tilde{\mathbf{h}}^T & \mathbf{0}_{1x3} & \mathbf{0}_{1x3} \\ \mathbf{0}_{1x3} & \Delta\tilde{\mathbf{h}}^T & \mathbf{0} \\ \mathbf{0}_{1x3} & \mathbf{0}_{1x3} & \Delta\tilde{\mathbf{h}}^T \end{bmatrix} \delta\mathbf{l} - \mathbf{L}\delta\tilde{\mathbf{h}}_p$.

30. A method according to Claim 21 wherein generating a truth reference field vector, $\tilde{\mathbf{h}}_i$, from inertial data and a three dimensional map of the earth's magnetic field comprises:

determining the earth's field vector in north/east/down frame components $\tilde{\mathbf{h}}^N$ based on one or more of a current latitude, longitude, altitude, and the map; and

transforming the earth's field vector into body coordinates to find errors in the "truth" source according to $\tilde{\mathbf{h}}_i = \mathbf{C}_L^B \mathbf{C}_N^L \tilde{\mathbf{h}}^N$, where $\mathbf{C}_L^B$ transforms a vector from the local-level frame (L-frame) to the body frame (B-frame) and is the transpose of the attitude direction cosine matrix **C**, and $\mathbf{C}_N^L$ accounts for the rotation in azimuth of the local-level frame with respect to north by the wander angle α, and is given by

$$\mathbf{C}_N^L = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

31. A method according Claim 30 further comprising taking partial differentials of $\tilde{\mathbf{h}}_i = \mathbf{C}_L^B \mathbf{C}_N^L \tilde{\mathbf{h}}^N$ according to

$$\begin{aligned} \delta\tilde{\mathbf{h}}_i &= \delta\mathbf{C}^T \mathbf{C}_N^L \tilde{\mathbf{h}}^N + \mathbf{C}^T \delta\mathbf{C}_N^L \tilde{\mathbf{h}}^N + \mathbf{C}^T \mathbf{C}_N^L \delta\tilde{\mathbf{h}}^N \\ &= \left[\mathbf{C}^T \{\gamma\}\right] \mathbf{C}_N^L \tilde{\mathbf{h}}^N + \mathbf{C}^T \left[-\{\varepsilon\} \mathbf{C}_N^L\right] \tilde{\mathbf{h}}^N + \mathbf{C}^T \mathbf{C}_N^L \delta\tilde{\mathbf{h}}^N \end{aligned},$$ where γ is the attitude error vector which represents the angular error of the L-frame relative to the B-frame, ε the angular position error vector which represents the angular error of the

L-frame relative to the earth frame (E-frame), and $\{\mathbf{v}\}$ represents the skew-symmetric matrix form of the enclosed vector $\mathbf{v}$, defined by $\{\mathbf{v}\} \equiv \begin{bmatrix} 0 & -v_z & v_y \\ v_z & 0 & -v_x \\ -v_y & v_x & 0 \end{bmatrix}$.

32. A method according Claim 31 further comprising:

estimating corrections using a Kalman filter implementing a "psi-angle" inertial error model where attitude error states are components of the angular error vector $\boldsymbol{\psi}$ defined as $\boldsymbol{\psi} = \boldsymbol{\gamma} - \boldsymbol{\varepsilon}$, which results in $\delta\tilde{\mathbf{h}}_i = \mathbf{C}^T \{\boldsymbol{\psi}\} \mathbf{C}_N^L \tilde{\mathbf{h}}^N + \mathbf{C}^T \mathbf{C}_N^L \delta\tilde{\mathbf{h}}^N$, or by taking a reversed order cross product, $\{\boldsymbol{\psi}\} \mathbf{C}_N^L \tilde{\mathbf{h}}^N = \boldsymbol{\psi} \times \left(\mathbf{C}_N^L \tilde{\mathbf{h}}^N\right) = -\left(\mathbf{C}_N^L \tilde{\mathbf{h}}^N\right) \times \boldsymbol{\psi} = -\left\{\mathbf{C}_N^L \tilde{\mathbf{h}}^N\right\} \boldsymbol{\psi}$, which results in $\delta\tilde{\mathbf{h}}_i = -\mathbf{C}^T \left\{\mathbf{C}_N^L \tilde{\mathbf{h}}^N\right\} \boldsymbol{\psi} + \mathbf{C}^T \mathbf{C}_N^L \delta\tilde{\mathbf{h}}^N$.

33. A method according to Claim 32 wherein determining a difference between a vector as measured by the magnetometer and the truth reference vector comprises stating the differences as $\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k$, where $\mathbf{z}_k$ is a 3x1 vector of measurements at time t_k, $\mathbf{H}_k$ is a 3xn measurement matrix, $\mathbf{x}_k$ is the state vector, and $\mathbf{v}_k$ is the measurement noise vector.

34. A method according to Claim 33 further comprising:

determining the measurement vector by subtracting $\delta\tilde{\mathbf{h}}_i = -\mathbf{C}^T \left\{\mathbf{C}_N^L \tilde{\mathbf{h}}^N\right\} \boldsymbol{\psi} + \mathbf{C}^T \mathbf{C}_N^L \delta\tilde{\mathbf{h}}^N$ from $\delta\tilde{\mathbf{h}} = \begin{bmatrix} \Delta\tilde{\mathbf{h}}^T & \mathbf{0}_{1x3} & \mathbf{0}_{1x3} \\ \mathbf{0}_{1x3} & \Delta\tilde{\mathbf{h}}^T & \mathbf{0} \\ \mathbf{0}_{1x3} & \mathbf{0}_{1x3} & \Delta\tilde{\mathbf{h}}^T \end{bmatrix} \delta\mathbf{l} - \mathbf{L}\delta\tilde{\mathbf{h}}_p$, resulting in

$$\begin{aligned} \mathbf{z} &= \tilde{\mathbf{h}} - \tilde{\mathbf{h}}_i = \delta\tilde{\mathbf{h}} - \delta\tilde{\mathbf{h}}_i + \mathbf{v} \\ &= \begin{bmatrix} \Delta\tilde{\mathbf{h}}^T & \mathbf{0}_{1x3} & \mathbf{0}_{1x3} \\ \mathbf{0}_{1x3} & \Delta\tilde{\mathbf{h}}^T & \mathbf{0} \\ \mathbf{0}_{1x3} & \mathbf{0}_{1x3} & \Delta\tilde{\mathbf{h}}^T \end{bmatrix} \delta\mathbf{l} - \mathbf{L}\delta\tilde{\mathbf{h}}_p + \mathbf{C}^T \left\{\mathbf{C}_N^L \tilde{\mathbf{h}}^N\right\} \boldsymbol{\psi} - \mathbf{C}^T \mathbf{C}_N^L \delta\tilde{\mathbf{h}}^N + \mathbf{v} \end{aligned},$$

where $\mathbf{v}$

represents a vector of uncorrelated measurement noise due to electromagnetic noise and random earth field modeling errors such as quantization.

35. A method according to Claim 34 further comprising generating a measurement mapping matrix $\mathbf{H}$ according to $\mathbf{H}=\begin{bmatrix}\mathbf{H}_{\psi} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{H}_{\delta\mathbf{l}} & \mathbf{H}_{\tilde{\mathbf{h}}_p} & \mathbf{H}_{\tilde{\mathbf{h}}^N}\end{bmatrix}$, where

$$\mathbf{H}_{\psi} = \mathbf{C}^T\left\{\mathbf{C}_N^L\tilde{\mathbf{h}}^N\right\}$$

$$\mathbf{H}_{\delta\mathbf{l}} = \begin{bmatrix}\Delta\tilde{\mathbf{h}}^T & \mathbf{0}_{1x3} & \mathbf{0}_{1x3} \\ \mathbf{0}_{1x3} & \Delta\tilde{\mathbf{h}}^T & \mathbf{0} \\ \mathbf{0}_{1x3} & \mathbf{0}_{1x3} & \Delta\tilde{\mathbf{h}}^T\end{bmatrix}.$$

$$\mathbf{H}_{\tilde{\mathbf{h}}_p} = -\mathbf{L}$$

$$\mathbf{H}_{\tilde{\mathbf{h}}^N} = -\mathbf{C}^T\mathbf{C}_N^L$$

36. A magnetic compass compensation unit for characterizing distortions in the earth's magnetic field caused by a vehicle, in which said unit is mounted, relative to an undisturbed magnetic field of the earth, said unit comprising a processor configured to receive measurements of the distorted magnetic field from a magnetometer, receive an orientation of the vehicle axes with respect to the earth at a time corresponding to each magnetometer measurement, receive undistorted earth magnetic field data for the vicinity of the vehicle relative to the earth at the time corresponding to each magnetometer measurement, and characterize the distortions utilizing the magnetic field measurements, the orientations of the vehicle, and the undistorted earth magnetic field data.

37. A magnetic compass compensation unit according to Claim 36, said processor receiving pitch and roll data from an attitude heading reference system.

38. A magnetic compass compensation unit according to Claim 36 wherein to characterize the distortions, said processor calculates correction coefficients based upon a magnetometer orientation with respect to the vehicle axes,

the magnetic field as measured by the magnetometer, magnetic inclination and declination data, and vehicle orientation.

39. A magnetic compass compensation unit according to Claim 38 wherein said processor is configured to calculate correction coefficients by estimating $\mathbf{L}_e$ and $\mathbf{H}_{pe}$ from the function $\mathbf{H}_{earth} = \mathbf{L}_e * \mathbf{H}_{meas} + \mathbf{H}_{pe}$ where, $\mathbf{H}_{earth}$ is a three-dimensional vector representing the undisturbed Earth's magnetic field, $\mathbf{H}_{meas}$ represents a three-dimensional vector for the disturbed Earth's magnetic field as measured by magnetometer, $\mathbf{L}_e$ is a three by three (3x3) matrix of magnetic correction coefficients, and $\mathbf{H}_{pe}$ is a three-dimensional vector of magnetic correction coefficients.

40. A magnetic compass compensation unit according to Claim 36 further configured to receive signals corresponding to magnetic sensors oriented in at least two different orientations.

41. A magnetic compass compensation unit according to Claim 36 further configured to receive signals corresponding to three magnetic sensors orthogonally oriented to one another.

42. A magnetic compass compensation unit according to Claim 36 further configured to determine vehicle orientation based upon a received GPS tracking angle.

43. A magnetic compass compensation unit according to Claim 36 further configured to determine a vehicle orientation based upon a true heading calculated utilizing GPS-augmented attitude and heading reference system algorithms.

44. A magnetic compass compensation unit according to Claim 36 further configured to determine a vehicle orientation based upon at least one of a true heading signal or a magnetic heading signal from an inertial reference system.

45. A magnetic compass compensation unit according to Claim 36 further configured to determine an orientation of the vehicle based upon a true heading received from a dual antenna GPS.

46. A magnetic compass compensation unit according to Claim 36 wherein to receive undistorted earth magnetic field data said processor accesses a table of magnetic inclination and declination data for a surface of the earth.

47. A magnetic compass compensation unit according to Claim 46 wherein said processor is configured to transform undistorted earth magnetic field data to correspond to axes of the magnetometer.

48. A processor programmed to generate a truth reference field vector, $\tilde{\mathbf{h}}_i$, from inertial data and a three dimensional map of the earth's magnetic field, determine a difference between a vector as measured by the magnetometer and the truth reference vector, and utilize the difference to estimate corrections to magnetometer model coefficients.

49. A processor according to Claim 48 wherein said processor comprises a Kalman filter for estimating corrections.

50. A processor according to Claim 48 wherein said processor is programmed to generate a truth reference field vector by relating the field measured by a magnetometer $\tilde{\mathbf{h}}_m$ to the true earth's field $\tilde{\mathbf{h}}$ according to $\tilde{\mathbf{h}}_m = \mathbf{M}\tilde{\mathbf{h}} + \tilde{\mathbf{h}}_p$, where M is a 3x3 magnetic permeability matrix and $\tilde{\mathbf{h}}_p$ is a 3x1 vector of field offset errors resulting from permanent magnetization.

51. A processor according to Claim 50 wherein said processor is programmed to solve for the true earth's field, $\tilde{\mathbf{h}}$, according to $\begin{aligned}\tilde{\mathbf{h}} &= \mathbf{M}^{-1}\left(\tilde{\mathbf{h}}_m - \tilde{\mathbf{h}}_p\right) \\ &= \mathbf{L}\left(\tilde{\mathbf{h}}_m - \tilde{\mathbf{h}}_p\right)\end{aligned}$, by assuming that the initial value of matrix **L** is equal to the identity matrix and the initial vector of field offset errors, $\tilde{\mathbf{h}}_p$, is zero.

52. A processor according to Claim 51 wherein said processor is programmed to:

provide a linearized error equation for the magnetometer-measured and compensated body-axis earth's field components according to
$$\begin{aligned}\delta\tilde{\mathbf{h}} &= \delta\mathbf{L}\left(\tilde{\mathbf{h}}_m - \tilde{\mathbf{h}}_p\right) - \mathbf{L}\delta\tilde{\mathbf{h}}_p \\ &= \delta\mathbf{L}\Delta\tilde{\mathbf{h}} - \mathbf{L}\delta\tilde{\mathbf{h}}_p\end{aligned}$$
, where $\Delta\tilde{\mathbf{h}}$ is defined as $\Delta\tilde{\mathbf{h}} = \tilde{\mathbf{h}}_m - \tilde{\mathbf{h}}_p$; and

generate a separate measurement equation for each field component from the linearized error equation according to $\delta\tilde{h}_x = \Delta\tilde{\mathbf{h}}^T\delta\mathbf{l}_{r1} - \mathbf{l}_{r1}^T\delta\tilde{\mathbf{h}}_p$, $\delta\tilde{h}_y = \Delta\tilde{\mathbf{h}}^T\delta\mathbf{l}_{r2} - \mathbf{l}_{r2}^T\delta\tilde{\mathbf{h}}_p$, and $\delta\tilde{h}_z = \Delta\tilde{\mathbf{h}}^T\delta\mathbf{l}_{r3} - \mathbf{l}_{r3}^T\delta\tilde{\mathbf{h}}_p$, where each equation is in the form of a row vector multiplied by a column vector of magnetometer parameter errors to provide scalars that are functions of the vectors.

53. A processor according to Claim 52 wherein said processor is programmed with magnetometer errors according to
$$\delta\tilde{\mathbf{h}} = \begin{bmatrix} \Delta\tilde{\mathbf{h}}^T & \mathbf{0}_{1x3} & \mathbf{0}_{1x3} \\ \mathbf{0}_{1x3} & \Delta\tilde{\mathbf{h}}^T & \mathbf{0} \\ \mathbf{0}_{1x3} & \mathbf{0}_{1x3} & \Delta\tilde{\mathbf{h}}^T \end{bmatrix}\delta\mathbf{l} - \mathbf{L}\delta\tilde{\mathbf{h}}_p.$$

54. A processor according to Claim 48 wherein to generate a truth reference field vector, $\tilde{\mathbf{h}}_i$, from inertial data and a three dimensional map of the earth's magnetic field, said processor is programmed to:

determine the earth's field vector in north/east/down frame components $\tilde{\mathbf{h}}^N$ based on one or more of a current latitude, longitude, altitude, and the map; and

transform the earth's field vector into body coordinates to find errors in the "truth" source according to $\tilde{\mathbf{h}}_i = \mathbf{C}_L^B\mathbf{C}_N^L\tilde{\mathbf{h}}^N$, where $\mathbf{C}_L^B$ transforms a vector from the local-level frame (L-frame) to the body frame (B-frame) and is the transpose of the attitude direction cosine matrix **C**, and $\mathbf{C}_N^L$ accounts for the rotation in azimuth of the local-level frame with respect to north by the wander angle α, and is given by

$$\mathbf{C}_N^L = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

55. A processor according Claim 54 wherein said processor is programmed to take partial differentials of $\tilde{\mathbf{h}}_i = \mathbf{C}_L^B \mathbf{C}_N^L \tilde{\mathbf{h}}^N$ according to $\begin{aligned}\delta\tilde{\mathbf{h}}_i &= \delta\mathbf{C}^T\mathbf{C}_N^L\tilde{\mathbf{h}}^N + \mathbf{C}^T\delta\mathbf{C}_N^L\tilde{\mathbf{h}}^N + \mathbf{C}^T\mathbf{C}_N^L\delta\tilde{\mathbf{h}}^N \\ &= \left[\mathbf{C}^T\{\boldsymbol{\gamma}\}\right]\mathbf{C}_N^L\tilde{\mathbf{h}}^N + \mathbf{C}^T\left[-\{\boldsymbol{\varepsilon}\}\mathbf{C}_N^L\right]\tilde{\mathbf{h}}^N + \mathbf{C}^T\mathbf{C}_N^L\delta\tilde{\mathbf{h}}^N\end{aligned}$, where $\boldsymbol{\gamma}$ is the attitude error vector which represents the angular error of the L-frame relative to the B-frame, $\boldsymbol{\varepsilon}$ the angular position error vector which represents the angular error of the L-frame relative to the earth frame (E-frame), and $\{\mathbf{v}\}$ represents the skew-symmetric matrix form of the enclosed vector $\mathbf{v}$, defined by $\{\mathbf{v}\} \equiv \begin{bmatrix} 0 & -v_z & v_y \\ v_z & 0 & -v_x \\ -v_y & v_x & 0 \end{bmatrix}$.

56. A processor according Claim 55 wherein said processor is programmed to estimate corrections using a Kalman filter which implements a "psi-angle" inertial error model, the inertial error model having attitude error states that are components of an angular error vector $\boldsymbol{\psi}$ defined as $\boldsymbol{\psi} = \boldsymbol{\gamma} - \boldsymbol{\varepsilon}$, which results in an error model of $\delta\tilde{\mathbf{h}}_i = \mathbf{C}^T\{\boldsymbol{\psi}\}\mathbf{C}_N^L\tilde{\mathbf{h}}^N + \mathbf{C}^T\mathbf{C}_N^L\delta\tilde{\mathbf{h}}^N$.

57. A processor according to Claim 56 wherein said processor is programmed to determine a difference between a vector as measured by the magnetometer and a truth reference vector comprises according to $\mathbf{z}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{v}_k$, where $\mathbf{z}_k$ is a 3x1 vector of measurements at time t_k, $\mathbf{H}_k$ is a 3xn measurement matrix, $\mathbf{x}_k$ is the state vector, and $\mathbf{v}_k$ is the measurement noise vector.

58. A processor according to Claim 57 wherein said processor is programmed to determine the measurement vector by subtracting

$\delta\tilde{\mathbf{h}}_i = -\mathbf{C}^T\left\{\mathbf{C}_N^L\tilde{\mathbf{h}}^N\right\}\boldsymbol{\psi} + \mathbf{C}^T\mathbf{C}_N^L\delta\tilde{\mathbf{h}}^N$ from $\delta\tilde{\mathbf{h}} = \begin{bmatrix} \Delta\tilde{\mathbf{h}}^T & \mathbf{0}_{1x3} & \mathbf{0}_{1x3} \\ \mathbf{0}_{1x3} & \Delta\tilde{\mathbf{h}}^T & \mathbf{0} \\ \mathbf{0}_{1x3} & \mathbf{0}_{1x3} & \Delta\tilde{\mathbf{h}}^T \end{bmatrix}\delta\mathbf{l} - \mathbf{L}\delta\tilde{\mathbf{h}}_p$, resulting in

$$\mathbf{z} = \tilde{\mathbf{h}} - \tilde{\mathbf{h}}_i = \delta\tilde{\mathbf{h}} - \delta\tilde{\mathbf{h}}_i + \mathbf{v}$$
$$= \begin{bmatrix} \Delta\tilde{\mathbf{h}}^T & \mathbf{0}_{1x3} & \mathbf{0}_{1x3} \\ \mathbf{0}_{1x3} & \Delta\tilde{\mathbf{h}}^T & \mathbf{0} \\ \mathbf{0}_{1x3} & \mathbf{0}_{1x3} & \Delta\tilde{\mathbf{h}}^T \end{bmatrix}\delta\mathbf{l} - \mathbf{L}\delta\tilde{\mathbf{h}}_p + \mathbf{C}^T\left\{\mathbf{C}_N^L\tilde{\mathbf{h}}^N\right\}\boldsymbol{\psi} - \mathbf{C}^T\mathbf{C}_N^L\delta\tilde{\mathbf{h}}^N + \mathbf{v},$$

where $\mathbf{v}$ represents a vector of uncorrelated measurement noise due to electromagnetic noise and random earth field modeling errors such as quantization.

59. A processor according to Claim 58 wherein said processor is programmed to generate a measurement mapping matrix $\mathbf{H}$ according to $\mathbf{H} = \begin{bmatrix} \mathbf{H}_\psi & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{H}_{\delta\mathbf{l}} & \mathbf{H}_{\tilde{\mathbf{h}}_p} & \mathbf{H}_{\tilde{\mathbf{h}}^N} \end{bmatrix}$, where

$$\mathbf{H}_\psi = \mathbf{C}^T\left\{\mathbf{C}_N^L\tilde{\mathbf{h}}^N\right\}$$
$$\mathbf{H}_{\delta\mathbf{l}} = \begin{bmatrix} \Delta\tilde{\mathbf{h}}^T & \mathbf{0}_{1x3} & \mathbf{0}_{1x3} \\ \mathbf{0}_{1x3} & \Delta\tilde{\mathbf{h}}^T & \mathbf{0} \\ \mathbf{0}_{1x3} & \mathbf{0}_{1x3} & \Delta\tilde{\mathbf{h}}^T \end{bmatrix}.$$
$$\mathbf{H}_{\tilde{\mathbf{h}}_p} = -\mathbf{L}$$
$$\mathbf{H}_{\tilde{\mathbf{h}}^N} = -\mathbf{C}^T\mathbf{C}_N^L$$